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EFFECT OF OPPOSITELY MOVING PLATES ON CONVECTIVE HEAT AND MASS TRANSFER THROUGH TWO IMMISCIBLE FLUIDS- FEA

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ABSTRACT

In this paper we present the novel of two immiscible fluids flow and the effect of heat and mass transfer along a oppositely moving vertical plates. The vertical channel is subjected to the transverse magnetic field and both plates are moving with constant velocity w_0 in opposite directions. The coupled governing equations of the flow and heat transfer with appropriate boundary conditions are solved using Finite Element method. The profiles of velocity, Micro rotation and temperature are studied for various parameters Grashof Number (Gr), magnetic field (H), Reynolds Number(R), Eckert Number (Ec), Material Parameter (K) and represented graphically. The Nusselt number and shear stress values are also analyzed.

KEYWORDS: Micropolar Fluid, Viscous Fluid, Vertical Channel, Moving Plates, Viscous Dissipation, MHD, FEM, Magnetic Field

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INTRODUCTION

The subject of two-fluid flow and heat transfer has been extensively studied due to its importance in chemical and nuclear industries. The design of two-fluid heat transport system for space application requires knowledge of heat and mass transfer processes and fluid mechanics under reduced gravity conditions. Identification of the two-fluid flow region and determination of the pressure drop, void fraction, quality reaction and two-fluid heat transfer coefficient are of great importance for the design of two-fluid systems. Lohrasbi and Sahai [3] studied two-phase MHD flow and heat transfer in a parallel plate channel with the fluid in one phase being electrically conducting. Malashetty and Leela [7] have analyzed the Hartmann flow characteristics of two fluids in horizontal channel. The study of two-phase flow and heat transfer in an inclined channel has been studied by Malashetty and Umavathi [5] and Malashetty et al [6].Micropolar fluids are non-Newtonian fluids with microstructures such as polymeric additives, colloidal suspensions, liquid crystals, etc. Eringen [1] developed the theory of Micropolar fluids, in which the microscopic effects arising from the local structure and the micro motions of the fluids elements are taken into account. The study of viscous dissipation is applicable to polymer technology involving the stretching of plastic sheets. The Problems of Micropolar fluid flow between two vertical plates (channel) are of great technical interest. A lot of attention has been given by many researchers. Suresh babu et.al [2] studied the heat transfer of micropolar and viscous fluids in a vertical channel. Sastry and Rao [11] have studied the effect of suction in the laminar flow of a Micropolar fluid in a channel. Bhargava and Rani [8] have

examined the convective heat transfer in Micropolar fluid flow between parallel plates. Stamonkovic et.al [9] studied the heat transfer of two immiscible fluids between horizontally moving plates.

Keeping in view of the wide area of practical importance of multi fluid flows as mentioned, the objective of this study to investigate the effect of heat and mass transfer in a vertical channel with oppositely moving plates.

Mathematical Formulation

The two infinite parallel plates are placed at $Y= - h_1$ and $Y= h_2$ along Y-direction initially as shown in Figure 1 and both plates are isothermal with different temperaturesT_1 and T_2 respectively. The distance from ($-h_1$ to 0) represents region 1 and distance from (0 to h_2) represents region 2 where the first region is filled with Micropolar fluid and the second is with viscous fluid. The fluid flow in the channel is due to buoyancy forces. The transport properties of both fluids are assumed to be constant.

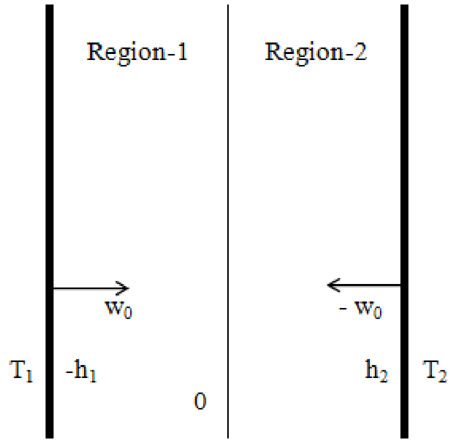

Figure 1: Physical Configuration

We consider the fluids to be incompressible and immiscible and the flow is steady laminar and fully developed.

The governing equations are

$$\frac{\partial U_1}{\partial Y}=0, \quad \frac{\partial U_2}{\partial Y}=0 \qquad \text{[Continuity]} \tag{1}$$

$$\rho_1=\rho_0[1-\beta_1(T_1-T_0)] \tag{2}$$

$$\rho_2=\rho_0[1-\beta_2(T_2-T_0)] \qquad \text{[State]} \tag{3}$$

$$(\mu_1+K)\frac{d^2U_1}{dY^2}+K\frac{dn}{dY}+\rho_1 g\beta_1(T_1-T_0)=\frac{\sigma B_0^{\,2}U_1}{\rho_1} \tag{4}$$

$$\mu_2\frac{d^2U_2}{dY^2}+\rho_2 g\beta_2(T_2-T_0)=\frac{\sigma B_0^{\,2}U_2}{\rho_2} \qquad \text{[Momentum]} \tag{5}$$

$$\gamma \frac{d^2 n}{dY^2} - K\left[2n + \frac{dU_1}{dY}\right] = 0$$ [Conservation of Angular Momentum] (6)

$$\frac{d^2 T_1}{dY^2} + \frac{\mu_1}{k_1}\left(\frac{dU_1}{dY}\right)^2 = 0$$

$$\frac{d^2 T_2}{dY^2} + \frac{\mu_2}{k_2}\left(\frac{dU_2}{dY}\right)^2 = 0$$ [Energy] (7)

$$\frac{d^2 C_1}{dY^2} = 0$$ (8)

$$\frac{d^2 C_2}{dY^2} = 0$$ [Diffusion] (9)

To solve the above system of equations, we use the boundary and interface conditions proposed by T.Arimen et al [10] as follows

$$U_1 = w_0 \;\; at \;\; Y = -h_1 \;,\; U_2 = -w_0 \;\; at \;\; Y = h_2$$

$$U_1(0) = U_2(0)$$

$$(\mu_1 + K)\frac{dU_1}{dY} + Kn = \mu_2 \frac{dU_2}{dY} \;,\; \frac{dn}{dY} = 0 \;\; at \;\; Y = 0 \;,$$

$$n = 0 \;\; at \, Y = -h_2$$

For the corresponding temperature boundary conditions it is assumed that the temperature and heat flows are continuous at the interface.

$$T = T_1 \;\; at \;\; Y = -h_1 \;, T = T_2 \;\; at \;\; Y = h_2$$

$$T_1(0) = T_2(0)$$

$$k_1 \frac{dT_1}{dY} = k_2 \frac{dT_2}{dY} \;\; at \, Y = 0$$

$$C = C_1 \;\; at \;\; Y = -h_1 \;, C = C_2 \;\; at \;\; Y = h_2$$

$$C_1(0) = C_2(0)$$

$$D_1 \frac{dC_1}{dY} = D_2 \frac{dC_2}{dY} \;\; at \, Y = 0$$

We assume that

$$\gamma = \left(\mu_1 + \frac{K}{2}\right) j$$

and $T_2 > T_1$

By introducing the following non dimensional variables:

$$y_1 = \frac{Y_1}{h_1},\ y_2 = \frac{Y_2}{h_2},\ u_1 = \frac{U_1}{U_0},\ u_2 = \frac{U_2}{U_0},\ \theta_1 = \frac{T_1 - T_0}{\Delta T},\ \theta_2 = \frac{T_2 - T_0}{\Delta T},\ N = \frac{h_1}{U_0} n,\ K' = \frac{\mu_1}{K}$$

The governing equations becomes

$$\frac{d^2 u_1}{dy^2} + \frac{K'}{1+K'}\frac{dN}{dy} + \frac{1}{1+K'}\left[\frac{Gr}{R}\theta_1\right] - \frac{1}{1+K'} M^2 u_1 = 0 \quad (10)$$

$$\frac{d^2 N}{dy^2} - \frac{2K'}{2+K'}\left(2N + \frac{du_1}{dy}\right) = 0 \quad (11)$$

$$\frac{d^2 u_2}{dy^2} + \left[\frac{Gr}{R}\theta_2\right]\frac{bm}{h\rho} - \frac{M^2 m}{h} u_2 = 0 \quad (12)$$

$$\frac{d^2 \theta_1}{dy^2} + \Pr Ec \left(\frac{du_1}{dy}\right)^2 = 0 \quad (13)$$

$$\frac{d^2 \theta_2}{dy^2} + \Pr Ec \frac{\alpha}{m}\left(\frac{du_2}{dy}\right)^2 = 0 \quad (14)$$

$$\frac{d^2 C_1}{dY^2} = 0 \quad (15)$$

$$\frac{d^2 C_2}{dY^2} = 0$$

Where

$$Gr = \frac{g\beta_1 \Delta T h_1^3}{v_1^2},\ R = \frac{U_0 h_1}{v_1}$$

$h = \frac{h_1}{h_2}$ (Channel width ratio), $m = \frac{\mu_1}{\mu_2}$ (viscosity ratio), $\alpha = \frac{k_1}{k_2}$ (Thermal conductivity ratio),

$\rho = \dfrac{\rho_1}{\rho_2}$ (Density ratio), $b = \dfrac{\beta_1}{\beta_2}$ (Thermal expansion coefficient ratio),

$M = \dfrac{\sigma \mu_e^2 H_0^2 h_1^2}{\mu_1}$, $\Pr = \dfrac{\mu_1 Cp}{k_1}$, $Ec = \dfrac{U_0^2}{Cp\ \Delta T}$

Subject to the boundary conditions:

$u_1(0) = u_2(0)$, $u_2 = -1 \ \ at \ \ y = 1$, $u_1 = 1 \ \ at \ \ y = -1$,

$$\frac{du_1}{dy} + \frac{K'}{1+K'} N = \frac{1}{mh(1+K')} \frac{du_2}{dy} \quad at \ \ y = 0$$

$\dfrac{dN}{dy} = 0 \ \ at \ y = 0$ $\quad N = 0 \ \ at \ \ y_1 = -1$

$\theta_1 = 1 \ \ at \ \ y_1 = -1$, $\theta_2 = 0 \ \ at \ \ y_2 = 1$, $\theta_1(0) = \theta_2(0)$

$$\frac{d\theta_1}{dy} = \frac{1}{h\alpha} \frac{d\theta_2}{dy} \quad at \ \ y = 0$$

$C_1 = 1 \ \ at \ \ Y = -1$, $C_2 = 0 \ \ at \ \ Y = 1$, $C_1(0) = C_2(0)$

$$\frac{dC_1}{dY} = \frac{1}{hD} \frac{dC_2}{dY} \quad at \ \ Y = 0$$

Solution of the Problem

The coupled governing equations are solved numerically using the regular Galerkin Finite Element method as given by J.N. Reddy [4]. For computational purpose each region is divided into 100 linear elements, each element is 3 nodded.

The shape functions at each node of a typical i^{th} element are the Langrange's interpolation polynomials given by

$$S_i^1 = \frac{\left(y - \left(\frac{2i-101}{100}\right)\right)\left(y - \left(\frac{2i-100}{100}\right)\right)}{\left(\frac{2i-102}{100} - \left(\frac{2i-101}{100}\right)\right)\left(\frac{2i-102}{100} - \left(\frac{2i-100}{100}\right)\right)}$$

$$S_i^2 = \frac{\left(y - \left(\frac{2i-102}{100}\right)\right)\left(y - \left(\frac{2i-100}{100}\right)\right)}{\left(\frac{2i-101}{100} - \left(\frac{2i-102}{100}\right)\right)\left(\frac{2i-101}{100} - \left(\frac{2i-100}{100}\right)\right)}$$

$$S_i^3 = \frac{\left(y-\frac{2i-101}{100}\right)\left(y-\left(\frac{2i-102}{100}\right)\right)}{\left(\frac{2i-100}{100}-\left(\frac{2i-102}{100}\right)\right)\left(\frac{2i-100}{100}-\left(\frac{2i-101}{100}\right)\right)}$$

The stiffness matrix equations corresponding to the governing equations (10) to (15) for i^{th} element are evaluated by using the following equations:

Region 1

$$S_i^p \frac{dU_{1i}^{\,j}}{dY} - \int_\Omega \frac{dU_{1i}^{\,j} S_i^j}{dY}\frac{dS_i^p}{dY}dY + \frac{k^1}{1+k^1}\left[N_i^j S_i^j S_i^p - \int_\Omega N_i^j S_i^j \frac{dS_i^p}{dY}dY\right]$$
$$+\frac{1}{1+k^1}\left[\frac{Gr}{R}\int_\Omega \theta_{1i}^{\,j} S_i^j S_i^p dY + \frac{Gc}{R}\frac{hD-Y}{1+hD}\int_\Omega S_i^p dY\right] - \frac{M^2}{1+k^1}\int_\Omega U_{1i}^{\,j} S_i^j dY = 0$$

$$S_i^p \frac{d\left(N_i^j S_i^j\right)}{dY} - \int_\Omega \frac{dS_i^j}{dY}\left(\frac{d\left(N_i^j S_i^j\right)}{dY}\right)dY - \frac{2K^1}{2+K^1}\left[2\int_\Omega \left(N_i^j S_i^j\right) S_i^p dY - S_i^p\left(U_{1i}^{\,j} S_i^j\right) + \int_\Omega \frac{dS_i^j}{dY}\left(U_{1i}^{\,j} S_i^j\right)dY\right] = 0$$

$$S_i^p \frac{d\left(\theta_{1i}^{\,j} S_i^j\right)}{dY} - \int_\Omega \left(\frac{dS_i^j}{dY}\frac{d\left(\theta_{1i}^{\,j} S_i^j\right)}{dY}\right)dY + \Pr Ec\int_\Omega \left(\frac{dU_{1i}^{\,j} S_i^j}{dY}\right)^2 S_i^p\, dY = 0$$

Region 2

$$S_i^p \frac{d\left(U_{2i}^{\,j} S_i^j\right)}{dY} - \int_\Omega \left(\frac{dS_i^j}{dY}\frac{d\left(U_{2i}^{\,j} S_i^j\right)}{dY}\right)dY + \frac{bm}{h\rho}\left[\frac{Gr}{R}\int_\Omega \left(\theta_{2i}^{\,j} S_i^j\right) S_i^p\, dY + \frac{hD}{1+hD}\frac{Gc}{R}\int_\Omega (1-Y) S_i^p dy\right]$$
$$-\frac{M^2 m}{h}\int_\Omega S_i^p\left(U_{2i}^{\,j} S_i^j\right)dY = 0$$

$$S_i^p \frac{d\left(\theta_{2i}^{\,j} S_i^j\right)}{dY} - \int_\Omega \left(\frac{dS_i^j}{dY}\frac{d\left(\theta_{2i}^{\,j} S_i^j\right)}{dY}\right)dY + \Pr Ec\frac{\alpha}{m}\int_\Omega S_i^j\left(\frac{dU_{2i}^{\,j} S_i^j}{dY}\right)^2 dY = 0$$

C_1 & C_2 are linear functions of Y.

Where Ω is the typical element region

$$\left(\frac{2i-102}{100}, \frac{2i-100}{100}\right)$$

These coupled governing equations are solved iteratively subject to the boundary conditions given in (14) until the desired accuracy of 10^{-5} is attained.

The Nusselt number and shear stress can be calculated at both walls by using the expressions

$$Nu_1 = \left[\frac{\partial \theta}{\partial Y}\right]_{Y=-1}, \quad Nu_2 = \left[\frac{\partial \theta}{\partial Y}\right]_{Y=1}, \quad St_1 = \left[\frac{\partial U_1}{\partial Y}\right]_{Y=-1}, \quad St_2 = \left[\frac{\partial U_2}{\partial Y}\right]_{Y=1}$$

RESULTS AND DISCUSSIONS

The results are depicted in the graphs from Figure 2 to Figure 19. The velocity profiles are from Figure 2 to Figure 7. The velocity is more in the mid region. The velocity is more in the viscous region for all variations than in micropolar region. The thermal buoyancy retards the flow (Figure 2). From Figure 3 it is evident that the increase in inertial force retards the flow in micropolar region significantly, but the variation in viscous region is negligible. The Lorentz force dominates the flow from Figure 4.The heat dissipation is inversely proportional to the flow. The effect of Ec is more significant in viscous region than the micropolar region. The effect of material parameter is minute on the velocity (Figure 6).

The variation of Micro rotation (N) with various parameters is exhibited from Figure 8 to Figure 13. The angular is momentum is reversal for all the variations due to the walls movement. The gradual variation of N is observed with Gr (Figure 8), R (Figure 9), H (Figure 10), Ec (Figure 11), K (Figure 12) and Gc (Figure 13). The Lorentz force is more predominant in enhancing the angular momentum where as all other forces considered are inversely proportional to the angular momentum.

All the temperature profiles are exhibited from figure 14 – 19. The effect of Gr (Figure 14), Gc (Figure 19),H (Figure 16) and Ec (Figure 17) on temperature is found significant. The buoyancy enhances the temperature. The Lorentz force retards the temperature. Obviously the heat dissipation enhances the temperature and it is more pronounced for Ec >0.003. The effects of Reynolds number and the micro rotation parameters on temperature are not much significant.

The Nusselt number and the shear stress is given in Table 1. The buoyancy reduces the heat transfer rate in micropolar fluid boundary and enhances at the viscous boundary. The reverse effect is observed for shear stress. The Lorentz force enhances the heat transfer rate at the micropolar fluid boundary but reduces at the viscous boundary. The viscous dissipation is also dominant in heat transfer rate and shear stress. Material property reduces the stress on the boundary.

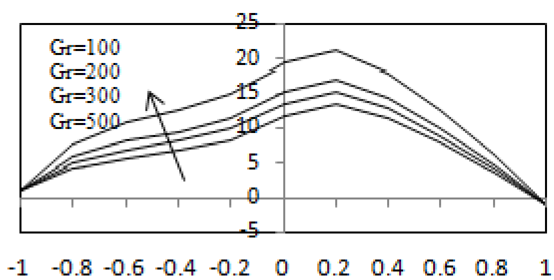

Figure 2: Variations of Velocity with Gr

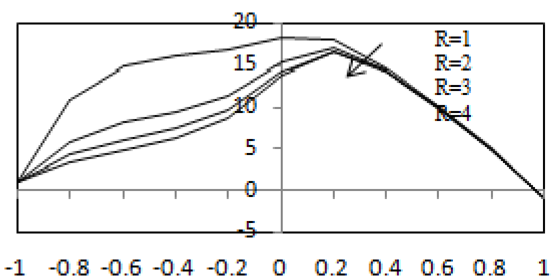

Figure 3: Variations of Velocity with R

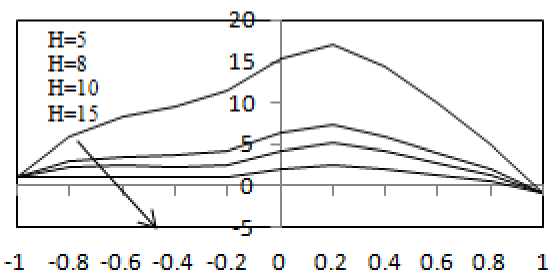

Figure 4: Variations of Velocity with H

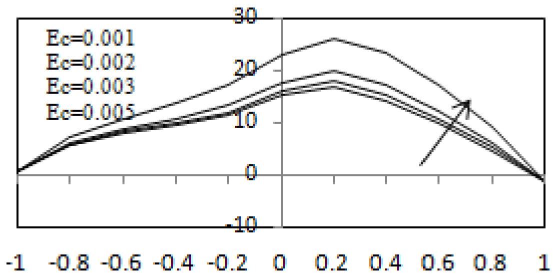

Figure 5: Variations of Velocity with Ec

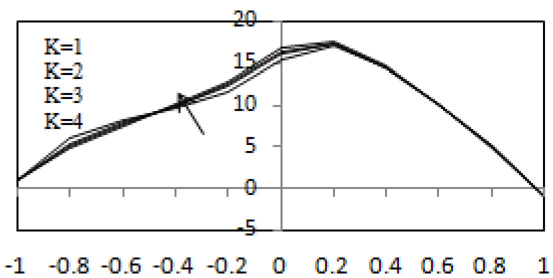

Figure 6: Variations of Velocity with K

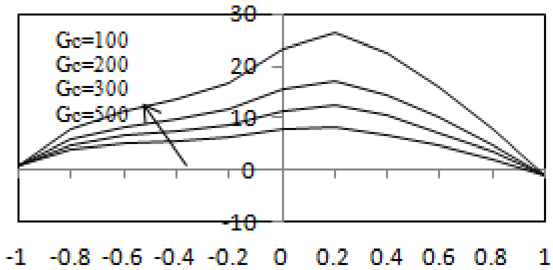

Figure 7: Variations of Velocity with Gc

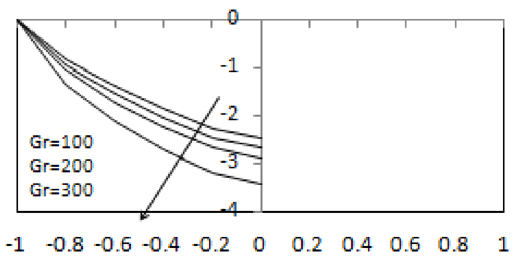

Figure 8: Variations of Micro Rotation with Gr

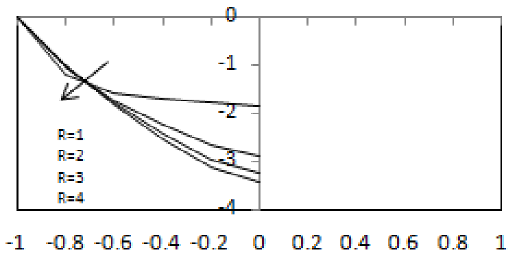

Figure 9: Variations of Micro Rotation with R

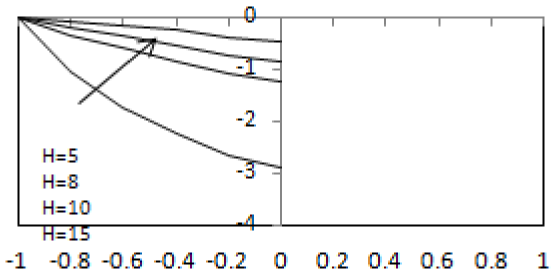

Figure 10: Variations of Micro Rotation with H

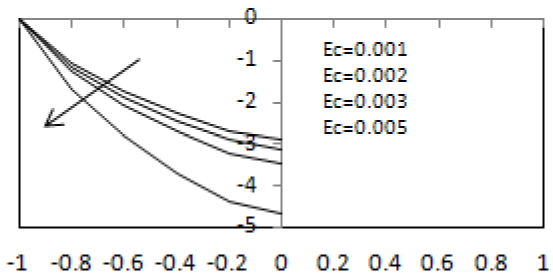

Figure 11: Variations of Micro Rotation with Ec

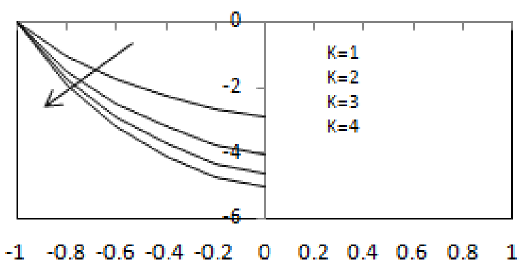

Figure 12: Variations of Micro Rotation with K

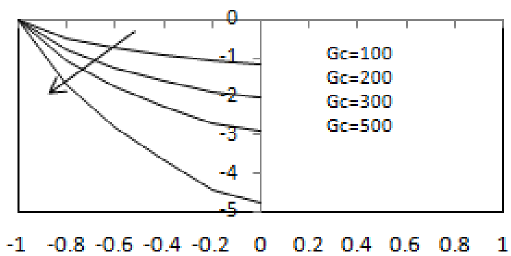

Figure 13: Variations of Micro Rotation with Gc

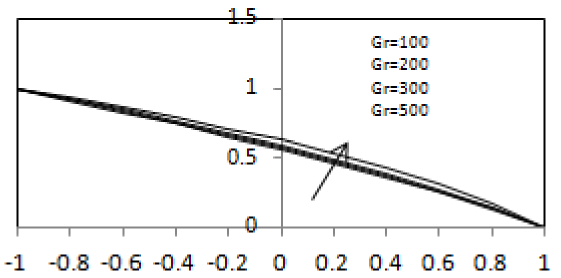

Figure 14: Variations of Temperature with Gr

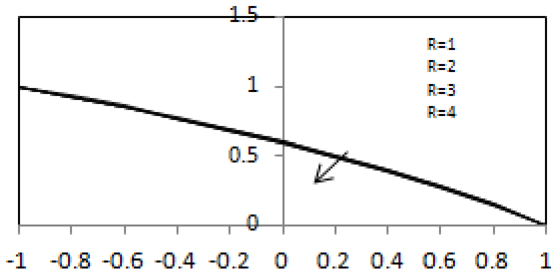

Figure 15: Variations of Temperature with R

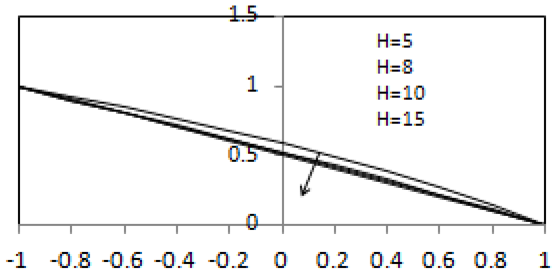

Figure 16: Variations of Temperature with H

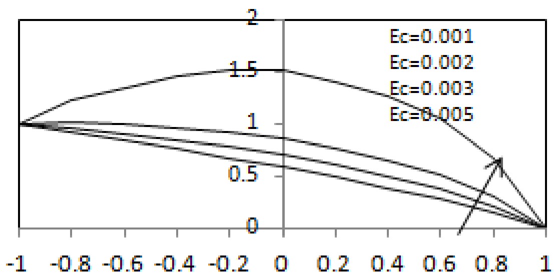

Figure 17: Variations of Temperature with Ec

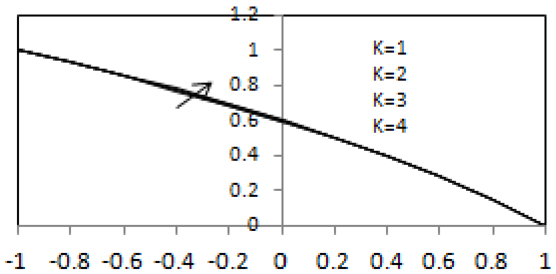

Figure 18: Variations of Temperature with K

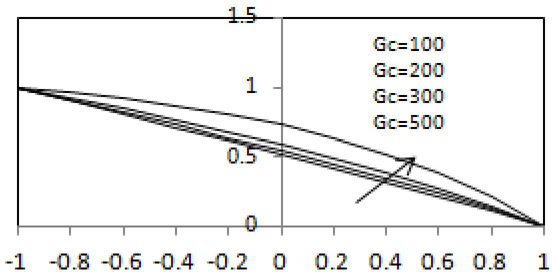

Figure 19: Variations of Temperature with Gc

Table 1: Nusselt Number and Shear Stress

Gr	R	H	Ec	K	St-I	St-II	Nu-I	Nu-II
100	2	5	0.001	1	36.2402	-28.7413	-0.395748	-0.663513
200	2	5	0.001	1	42.3718	-31.408	-0.361873	-0.702001
300	2	5	0.001	1	48.5982	-34.352	-0.319367	-0.748942
300	1	5	0.001	1	84.3885	-34.5898	-0.0843623	-0.755735
300	3	5	0.001	1	36.8886	-34.4316	-0.347081	-0.761117
300	4	5	0.001	1	31.0417	-34.5025	-0.351712	-0.770067
300	2	8	0.001	1	32.1439	-16.1166	-0.458378	-0.556303
300	2	10	0.001	1	26.2391	- 11.321	-0.476314	-0.534776
300	2	15	0.002	1	17.2485	-5.43871	-0.476903	-0.5424
300	2	5	0.003	1	50.6399	-40.785	0.185761	-1.52488
300	2	5	0.001	2	42.738	-34.4623	-0.321857	-0.75571
300	2	5	0.001	3	39.4375	-34.5435	-0.32035	-0.761079

CONCLUSIONS

- The velocity increases drastically in the mid region and more in viscous region.
- The temperature is more in Micropolar region than the viscous region.

- The viscous dissipation enhances the heat transfer on the boundary Y= -h_1 and depreciates the heat transfer on the boundary at Y= h_2.The reverse effect has been observed due to the magnetic field.
- The viscous dissipation depreciates the stress on both boundaries. The reverse effect has been observed due to the magnetic field.

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APPENDICES

Nomenclature

U_1 :velocity in the region 1 U_2 :velocity in the region 2 U_0 :Average Velocity T_1:Temperature of the plate at $y=-h_1$ T_2 :Temperature of the plate at $y=h_2$ T_0 :Average temperature. k_1 :Thermal conductivity in Region 1	Nu_1 :Nusselt number at $y=-1$ Nu_2 :Nusselt number at $y=1$ St_1 : shear stress at $y=-1$ St_2 : shear stress at $y=1$ <u>Greek letters:</u> ρ_1 :density of fluid in Region 1 ρ_2 :density of fluid in Region 2 β_1 :Coefficient of Thermal expansion in Region 1

k_2 :Thermal conductivity in Region 2 g :Acceleration due to gravity n :Micro rotation parameter K :Vertex viscosity H_0 :Magnetic Field Intensity Ec :Eckert number Gr :Groshof number R : Reynolds Number N :Micro rotation number h :channel width ratio, m :viscosity ratio b :Thermal expansion coefficient ratio B_0 :Magnetic induction	β_2 :Coefficient of Thermal expansion in Region 2 μ_1 :Viscosity of fluid in Region 1 μ_2 :Viscosity of fluid in Region 2 σ : electrical conductivity γ :Spin gradient γ_1 :Dynamic viscosity of Micropolar fluid μ_e :Magnetic Field Permeability α :Thermal conductivity ratio ρ :density ratio $\Pr$:Prandtl number